1. THE INTEGRATED OPTIMIZATION PROBLEM.

$$\pi^m = [p - c - \Phi(s)]\, D(p,s)$$

$$\partial \pi / \partial p = D(p^m, s^m) \qquad + [p^m - c - \Phi(s^m)]\, \partial D / \partial p = 0$$

$$\partial \pi / \partial s = -\Phi'(s)\, D(p^m, s^m) + [p^m - c - \Phi(s^m)\, \partial D / \partial s = 0$$

NOW CONSIDER THE PROBLEM FOR THE UNINTEGRATED RETAILER.

$$\pi = [p - p_w - \Phi(s)]\, D(p,s)$$

$$\text{s.t. } D(p^m, s^m) = q$$

$$\mathcal{L} \equiv [p - p_w - \dot{\Phi}(s)]\, D(p,s) - \lambda [q - D(p,s)]$$

$$\partial \mathcal{L} / \partial p = D(p,s) + [p - p_w - \Phi(s) + \lambda]\, \partial D / \partial p = 0$$

$$\partial \mathcal{L} / \partial s = -\Phi(s)\, D(p,s) + [p - p_w - \Phi(s) + \lambda]\, \partial D / \partial s = 0$$

$$\partial L / \partial \lambda = q - D(p,s) = 0 \Rightarrow D(p,s) = D(p^m, s^m)$$

$$\Rightarrow -\frac{1}{\dot{\Phi}(s)} = \frac{\partial D / \partial p}{\partial D / \partial s}$$

THIS SAME CONDITION IS IMPLIED BY THE THE INTEGRATED FOC. SO WITH THE SAME q WE WILL GET THE SAME p, s CHOSEN. ALL PROFITS ARE EXTRACTED WITH $p_? = p^m - \Phi(s^m)$ ALSO SEE TIROLE ANSWER FOR EXERCISE 4.3

2. (a) IF THE REGULATOR ONLY HAS THE LOW COST FIRM PRODUCE THEN ONLY THE LOW COST PRODUCERS (IR) CONSTRAINT NEEDS TO HOLD AND HE CAN INDUCE THE EFFICIENT OUTPUT FROM THE FIRM. HENCE,

$$P_L = c'(D(P_L)|L)$$
$$T_L = P_L D(P_L) - c(D(P_L)|L) - \underline{\Pi}$$

WHEN THE REGULATOR ONLY HAS THE LOW COST FIRM PRODUCE IT'S PAY OFF IS

$\phi_L \lambda S(P_L)$ WHERE $S(\cdot)$ IS THE SURPLUS FROM THE FIRM'S OUTPUT.

IF THIS PAYOFF IS HIGHER THEN THE MAXIMUM PAYOFF TO THE REGULATOR FROM HAVING BOTH TYPES PRODUCE THEN IT IS BETTER HAVING ~~BOTH~~ ONLY THE LOW TYPE PRODUCE.

(b) IF LOW COST IS ANNOUNCED, THEN HAVE THE FIRM PRODUCE EFFICIENTY AND TAX T_L. IF HIGH COST IS ANNOUNCED, THE AUDIT WITH PROB. s AND TAX T_{Hi} WHERE $i \in \{L, H\}$ IS THE RESULT OF THE AUDIT. IF THE IS NO AUDIT, TAX ASSUMING THAT THE AUDIT COINCIDES WITH THE ANNOUNCEMENT.

TO SEE THAT THIS SATISFIES (IC) AND (IR) CONSTRAINTS NOTE THAT

$$(IC) \Leftrightarrow \pi(x_L|L) - T_L \geq \pi(x_H|L) - (1-s)T_{HH} - s[\psi_{LL}T_{HL} + \psi_{HL}T_{HH}] \quad (1)$$

$$\pi(x_H|H) - (1-s)T_{HH} - s[\psi_{LH}T_{HL} + \psi_{HH}T_{HH}] \geq \pi(x_L|H) - T_L \quad (2)$$

$$(IR) \quad \pi(x_L|L) - T_L \geq \underline{\pi} \quad (3)$$

$$\pi(x_H|H) - (1-s)T_{HH} - s[\psi_{LH}T_{HL} + \psi_{HH}T_{HH}] \geq \underline{\pi} \quad (4)$$

FULL EXTRACTION FROM THE L TYPE IMPLIES (3) HOLDS WITH EQUALITY AND (2) MUST BE SATISFIED IF (4) IS SATISFIED. SO THAT MEANS THAT

$T_L = \pi(x_L|L) - \underline{\pi}$ AND (1) AND (4) IMPLIES (2)+(3) SO FOR $T_L = \pi(x_L|L) - \underline{\pi}$ IF T_{HL} AND T_{HH} CAN BE FOUND THAT SATISFY (1) AND (4), THEN AN OPTIMAL SOLUTION EXISTS.

3. THE REGULATOR SEEKS TO PICK A MENU OF CONTRACTS $(x(\theta), T(\theta))$ THAT SOLVES

$$\max_{x(\theta),T(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \lambda \left[\int_0^{x(\theta)} (\theta - \xi)\, d\xi - (\theta - x(\theta))\, x(\theta) + T(\theta) \right] + (1-\lambda)\pi(\theta) \right\} f(\theta)\, d\theta$$

s.t. (IC) AND (IR) CONSTRAINTS.

WHERE $\pi(\theta)$ IS THE EXPECTED PROFIT OF THE MONOPOLIST GIVEN IT IS TYPE θ.

$$\forall \theta, \tilde{\theta}$$

$$(IC) \Rightarrow \pi(\theta) \geq (\theta - x(\tilde{\theta}))\, x(\tilde{\theta}) - \tfrac{1}{2} x(\tilde{\theta})^2 - T(\tilde{\theta})$$

$$\Rightarrow \pi(\theta) - \pi(\tilde{\theta}) \geq [\theta - x(\tilde{\theta})]\, x(\tilde{\theta}) - [\tilde{\theta} - x(\tilde{\theta})]\, x(\tilde{\theta})$$

$$= [\theta - \tilde{\theta}]\, x(\tilde{\theta})$$

$$\Rightarrow [\theta - \tilde{\theta}]\, x(\theta) \geq \pi(\theta) - \pi(\tilde{\theta}) \geq [\theta - \tilde{\theta}]\, x(\tilde{\theta})$$

$$\Rightarrow \pi(\theta) - \pi(\tilde{\theta}) = \int_{\tilde{\theta}}^{\theta} x(\xi)\, d\xi$$

THE LOWEST PROFIT WOULD GO TO A MONOPOLIST OF TYPE $\underline{\theta}$. HENCE, THE REGULATOR SHOULD EXTRACT ALL OF TYPE $\underline{\theta}$'S PROFITS. NORMALIZING RESERVATION UTILITY TO ZERO, $\pi(\underline{\theta}) = 0$ AND $\pi(\theta) = \int_{\underline{\theta}}^{\theta} x(\xi)\, d\xi$.

$$\int_{\underline{\theta}}^{\bar{\theta}} \pi(\theta) f(\theta)\, d\theta = \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} x(\xi)\, d\xi\, f(\theta)\, d\theta$$

$$= F(\theta) \int_{\underline{\theta}}^{\theta} x(\xi)\, d\xi \Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) F(\theta)\, d\theta$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} [1 - F(\theta)]\, x(\theta)\, d\theta$$

$$\Pi(\theta) = [\theta - x(\theta)]\, x(\theta) - \tfrac{1}{2} x(\theta)^2 - \int_{\underline{\theta}}^{\theta} x(\xi)\,d\xi$$

SO THE REGULATORS OBJECTIVE FUNCTION IS

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ \lambda \left[\int_0^{x(\theta)} [\theta - \xi]\,d\xi - \tfrac{1}{2} x(\theta)^2 \right] - (2\lambda - 1) \frac{1 - F(\theta)}{f(\theta)}\, x(\theta) \right\} f(\theta)\,d\theta$$

APPLYING POINTWISE OPTIMIZATION, THE FIRST ORDER CONDITION FOR THE OPTIMAL $x(\theta)$ IS

$$[\theta - x(\theta)] - x(\theta) - \frac{2\lambda - 1}{\lambda} \, \frac{1 - F(\theta)}{f(\theta)} = 0$$

IF OUTPUT IS EFFICIENT, THEN $\theta = 2x(\theta)$. OR
$x(\theta) = \theta/2$.

$$x(\theta) = \tfrac{1}{2}\theta - \tfrac{1}{2} \, \frac{2\lambda - 1}{\lambda} \, \frac{1 - F(\theta)}{f(\theta)}$$