ANSWERS TO PROBLEM SET #2

1. AS LONG A BOTH FIRMS DO NOT SET PRICES ABOVE THE MONOPOLY LEVEL ALL CONSUMERS WILL BUY. THE QUESTION ALSO SAYS TO ASSUME THAT BOTH FIRMS MAKE SALES. HENCE, THERE WILL BE A CONSUMER WHO IS INDIFFERENT BETWEEN BUYING FROM THE TWO FIRMS. θ^* IS INDIFFERENT IF

$$P_0 + t\theta^* = P_1 + t(1-\theta^*)$$
$$\Rightarrow \quad \theta^* = \frac{P_1 - P_2}{2t} + \frac{1}{2}$$

FIRM 0'S PROFITS:

$$\pi_0 = [P_0 - C_0]\left[\frac{P_1 - P_0}{2t} + \frac{1}{2}\right]$$

$$\frac{\partial \pi_0}{\partial P_0} = \frac{P_1 - P_0}{2t} + 1/2 - \frac{1}{2t}[P_0 - C_0] = 0$$

$$P_1 - 2P_0 + t + C_0 = 0$$

$$P_0 = \frac{P_1 + t + C_0}{2}$$

FIRM 1'S PROFITS:

$$\pi_\phi = [P_1 - C_1]\left[\frac{P_0 - P_1}{2t} + \frac{1}{2}\right]$$

$$\frac{\partial \pi_1}{\partial P_1} = P_0 - 2P_1 + t + C_1 = 0$$

$$\frac{P_1 + t + C_0}{2} - 2P_1 + t + C_1 = 0$$

$$P_1 + t + C_0 - 4P_1 + 2t + 2C_1 = 0$$

$$P_1^* = \frac{C_0 + 2C_1}{3} + t$$

$$P_0^* = \frac{C_1 + 2C_0}{3} + t$$

2. FIND THE LOCATION OF THE INDIFFERENT CONSUMER.

$$\bar{s} - p_0 - t_0\theta^* = \bar{s} - p_1 - t_1(1-\theta^*)$$

$$\Rightarrow \quad p_0 + t_0\theta^* = p_1 + t_1(1-\theta^*)$$

$$\Rightarrow \quad (t_0 + t_1)\theta^* = p_1 - p_0 + t_1$$

$$\theta^* = \frac{p_1 - p_0}{t_0 + t_1} + \frac{t_1}{t_0 + t_1} \qquad \boxed{\text{FOR } t_0 + t_1 > 0}$$

$$\pi_0 = p_0\left[\frac{p_1 - p_0}{t_0+t_1} + \frac{t_1}{t_0+t_1}\right]$$

$$\frac{\partial\pi_0}{\partial p_0} = \frac{p_1 - 2p_0 + t_1}{t_0 + t_1} = 0 \Rightarrow p_0 = \frac{p_1 + t_1}{2}$$

$$\pi_1 = p_1\left[\frac{p_0 - p_1}{t_0+t_1} + \frac{t_0}{t_0+t_1}\right]$$

$$\frac{\partial\pi_1}{\partial p_1} = p_0 - 2p_1 + t_0 = 0$$

$$p_0^* = \frac{t_0 + 2t_1}{3} \qquad p_1^* = \frac{t_1 + 2t_0}{3}$$

FOR $t_0 = t_1 = 0$, WE HAVE HOMOGENEOUS GOOD BERTRAND COMPETITION WHERE BOTH FIRMS EARN ZERO PROFITS.

NOW SUPPOSE FIRM 0 DEVIATES FROM $t_0 = 0$ AND SETS $t_0 > 0$. THEN WE HAVE $p_0^* = \frac{1}{3}t_0$ $p_1^* = \frac{2}{3}t_0$ AND $\theta^* = 1/3$. NOTE THAT THIS DEVIATION EARNS FIRM 0 POSITIVE PROFITS, SO $t_0 = t_1 = 0$ CANNOT BE AN EQUILIBRIUM.

3. WE NEED TO FIND THE OPTIMAL CONTRACT FOR FIRM 0 GIVEN A CONTRACT BY FIRM 1.

$$U^i(\theta) = u(q^i(\theta), \theta, v) - p^i(\theta)$$

WE ALSO KNOW THAT

$$U^i(\theta) = U^i(\bar{\theta}_i) - \int_\theta^{\bar{\theta}_i} u_\theta(q^i(\xi), \xi, v)\,d\xi$$

$$= U^i(\bar{\theta}_i) + \int_\theta^{\bar{\theta}_i} q^i(\xi)\,d\xi$$

HENCE,

$$p^i(\theta) = (5-\theta)q^i(\theta) - \int_\theta^{\bar{\theta}_i} q^i(\xi)\,d\xi - U^i(\bar{\theta}_i)$$

EXPECTED PROFITS ARE

$$\int_0^{\bar{\theta}_i} \left[p^i(\theta) - \tfrac{1}{2}q^i(\theta)^2\right]d\theta$$

$$= \int_0^{\bar{\theta}_i} \left[(5-\theta)q^i(\theta) - \underbrace{\int_\theta^{\bar{\theta}_i} q^i(\xi)\,d\xi}\, - U^i(\bar{\theta}_i) - \tfrac{1}{2}q^i(\theta)^2\right]d\xi$$

By Integration by parts

$$\boxed{\begin{aligned}\int_0^1 \int_\theta^{\bar{\theta}_i} q^i(\xi)\,d\xi\,d\theta &= \int_\theta^{\bar{\theta}_i} q^i(\xi)\,d\xi\,\theta\,\Big|_0^{\bar{\theta}_i} + \int_0^{\bar{\theta}_i} q^i(\theta)\,\theta\,d\theta \\ &= \int_0^{\bar{\theta}_i} q^i(\theta)\,\theta\,d\theta\end{aligned}}$$

$$= \int_0^{\bar{\theta}_i} \left[(5-2\theta)q^i(\theta) - \tfrac{1}{2}q^i(\theta)^2 - U^{-i}(1-\bar{\theta}_i)\right]d\theta$$

FIRST USE FIRST-ORDER CONDITION TO SOLVE FOR q^i.

$$\Rightarrow 5 - 2\theta - q^i(\theta) = 0 \Rightarrow q^i(\theta) = 5 - 2\theta$$

NOW USE THE FIRST-ORDER CONDITION TO SOLVE FOR [redacted] $U^i(\bar{\theta}_i)$. [SYMMETRY WOULD IMPLY $\bar{\theta}_i = 1/2$.]

$$(5-2\bar{\theta})^2 - \tfrac{1}{2}(5-2\bar{\theta})^2 - U^{-i}(1-\bar{\theta}) - q^{-i}(1-\bar{\theta}_i)\bar{\theta}_i = 0$$

[NOTE: $U^{-i}(1-\bar{\theta}) = U^{-i}(\bar{\theta}_{-i}) + \int_{1-\bar{\theta}_i}^{\bar{\theta}_{-i}} q^{-i}(\varsigma)\, d\varsigma$

AND $\dfrac{dU^{-i}(1-\bar{\theta})}{d\bar{\theta}} = q^{-i}(1-\bar{\theta}_i)$]

APPLY SYMMETRY : $\bar{\theta} = 1/2$ AND $q^{-i}(1/2) = q^i(1/2)$

$$\Rightarrow \tfrac{1}{2}4^2 - \tfrac{1}{2}4 - U^{-i}(1/2) = 0$$

$$\Rightarrow U^i(\bar{\theta}) = U^{-i}(1/2) = 6$$