Problem Set #1

Economics 661 Spring 2006

1. Consider the linear highway model discussed in class. Specifically, suppose that the preferences of a consumer located at $x \in [0,1]$ are characterized by the following utility function

$$U = \begin{cases} \overline{s} - t|x - x^*| - p^*, & \text{if a purchase is made from location } x^* \in [0,1] \\ 0, & \text{if no purchase is made} \end{cases}$$

where $\overline{s}, t > 0$ and p^* denotes the price at location x^*. Suppose consumers are uniformly distributed on the unit interval and the monopolist's marginal cost is zero.

(a) What is the minimum value for $\overline{s}$ for which it is still profit maximizing for a monopolist located at 0 to sell to all consumers?

(b) Suppose that $\overline{s}$ is at or above the level you found in Part (a). Find the profit maximizing price for a monopolist as a function of his location. Said another way, for every possible single location of the monopolist on the unit interval, find the profit maximizing price.

(c) Suppose $\overline{s} = t$ and the monopolist has an established location at 0. The fixed cost of opening a new location at 1 is $f \in R_{++}$. Find the set of fixed costs for which the monopolist will open a new location at 1, and find the set of fixed costs for which a social planner will open a new location at 1. Provide some intuition for the difference.

(d) Suppose that $t = 2$ for consumers $x \in [0, \frac{1}{4}]$ and $t = 1$ for consumers $x \in [\frac{1}{4}, 1]$. When $\overline{s} = 5$, find the set of optimal locations seperately for a profit maximizing a monopolist and for a transportation cost minimizing social planner. Why are they different?

2. Now suppose instead of a line, consider a hexagon model of spacial differentiation. 1/6th of the population lives at each of the six points. The distance between each of the points is 1/6th. In order to get from one point to another, consumers must travel along the boundary of the hexagon rather than across its interior. The population at 5 of the 6 points have a transportation cost of $t = 1$, and the population at one of the points have a transportation cost of $t = 2$. Suppose only a single location must be chosen and $\overline{s} = 5$. Compare the set of optimal locations for a monopolist and for a transportation cost minimizing social planner. What insights can be drawn?

3. Let $\theta \in [0, 1/2]$ denote the type of consumer which is also equal to the probability of a loss L. θ is distributed by

$$F(\theta) = \begin{cases} 0, & \text{for } \theta < 0 \\ 2\theta, & \text{for } \theta \in [0, 1/2] \\ 1 & \text{for } \theta > 1/2. \end{cases}$$

Assume that consumers are willing to pay $V(\theta) = \alpha\theta$ for insurance to cover the loss L. Consumers are risk-averse so $\alpha > L$. Assuming that the insurance market is "competitive" (i.e., zero profit), find the set of consumers for which insurance will be offered when insurers cannot observe an individuals type. [Be careful: There was a slight mistake in the conditions I wrote on the board in class.]